

$$\begin{aligned}
\sqrt{(\sin x + \cos x)} &= \cos 2x \\
\sin x + \cos x &= \cos^2 2x \\
\sin x + \cos x &= (\cos^2 x - \sin^2 x)^2 \\
\sin x + \cos x &= ((\sin x + \cos x)(\cos x - \sin x))^2 \\
\sin x + \cos x - ((\sin x + \cos x)(\cos x - \sin x))^2 &= 0 \\
(\sin x + \cos x) (1 - (\sin x + \cos x)(\cos x - \sin x)^2) &= 0 \\
\sin x + \cos x &= 0 \\
\sqrt{2} \sin(x + \pi/4) &= 0 \\
x + \pi/4 &= \pi k \\
x &= \pi k - \pi/4 \\
1 - (\sin x + \cos x)(\cos x - \sin x)^2 &= 0 \\
1 - (\sin x + \cos x)(\cos^2 x - 2\sin x \cos x + \sin^2 x) &= 0 \\
1 - (\sin x + \cos x)(1 - 2\sin x \cos x) &= 0 \\
\sin x + \cos x &= t \\
t^2 &= \sin^2 x + 2\sin x \cos x + \cos^2 x = 2\sin x \cos x + 1 \\
2\sin x \cos x &= t^2 - 1 \\
1 - t(1 - (t^2 - 1)) &= 0 \\
1 - t(2 - t^2) &= 0 \\
1 + t^3 - 2t &= 0 \\
t^3 - 2t + 1 &= 0 \\
t &= 1 \\
\begin{matrix} 1 & 0 & -2 & 1 \\ 1 & 1 & 1 & -1 & 0 \end{matrix} \\
t^2 + t - 1 &= 0 \\
D &= 1 + 4 = 5 \\
t_{1,2} &= (-1 \pm \sqrt{5})/2 \\
\sin x + \cos x &= 1 \\
\sqrt{2} \sin(x + \pi/4) &= 1 \\
x + \pi/4 &= \pi/4 + 2\pi k \\
x &= 2\pi k \\
x &= \pi - 2\pi/4 + 2\pi k \\
x &= 3\pi/4 + 2\pi k
\end{aligned}$$

$$\begin{aligned}
\cos 2x &\geq 0 \\
-\pi/2 + 2\pi k &\leq 2x \leq \pi/2 + 2\pi k \\
-\pi/4 + \pi k &\leq x \leq \pi/4 + \pi k
\end{aligned}$$

$$\begin{aligned}
\sqrt{1 - \cos 2x} &= \sin 2x \\
\sqrt{2(1 - \cos 2x)/2} &= \sin 2x \\
\sqrt{2\sin^2 x} &= \sin 2x \\
\sqrt{2} |\sin x| &= \sin 2x
\end{aligned}$$

$$\begin{aligned}
\sin x &\geq 0 \\
\sqrt{2} \sin x &= \sin 2x \\
\sin x (\sqrt{2} - 2\cos x) &= 0 \\
\sin x &= 0 \\
x &= \pi k \\
2\cos x &= \sqrt{2} \\
\cos x &= \sqrt{2}/2 \\
x &= \pm \pi/4 + 2\pi k \\
x &= \pi/4 + 2\pi k
\end{aligned}$$

$$\begin{aligned}
\sin x &< 0 \\
-\sqrt{2} \sin x &= \sin 2x \\
-\sin x (\sqrt{2} + 2\cos x) &= 0 \\
-\sin x &= 0 \\
x &= \pi k \\
\sqrt{2} + 2\cos x &= 0 \\
\cos x &= -\sqrt{2}/2 \\
x &= \pm 3\pi/4 + 2\pi k \\
x &= -3\pi/4 + 2\pi k
\end{aligned}$$

**Ответ  $\pi k; \pi/4 + 2\pi k; -3\pi/4 + 2\pi k$**

$$\begin{aligned}
\sqrt{2} \sin(x + \pi/4) &= (-1 + \sqrt{5})/2 \\
\sin(x + \pi/4) &= (-1 + \sqrt{5})\sqrt{2}/4 \\
x + \pi/4 &= \arcsin((-1 + \sqrt{5})\sqrt{2}/4) + 2\pi k \\
x &= \arcsin((-1 + \sqrt{5})\sqrt{2}/4) - \pi/4 + 2\pi k \\
x &= 3\pi/4 - \arcsin((-1 + \sqrt{5})\sqrt{2}/4) + 2\pi k
\end{aligned}$$

с минусом не подойдёт

Ответ до ОДЗ

$$\begin{aligned}
x &= 3\pi/4 - \arcsin((-1 + \sqrt{5})\sqrt{2}/4) + 2\pi k; \arcsin((-1 + \sqrt{5})\sqrt{2}/4) - \pi/4 + 2\pi k; x = 2\pi k; \\
x &= 3\pi/4 + 2\pi k
\end{aligned}$$

Ответ после ОДЗ

$$x = \arcsin((-1 + \sqrt{5})\sqrt{2}/4) - \pi/4 + 2\pi k; x = 2\pi k; x = 3\pi/4 + 2\pi k$$

